XXXIV. COMPUTATION RESEARCH*

Research Staff

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RESEARCH OBJECTIVES

This group provides a general programming service for Laboratory members who use computers in their research and also acts as liaison with the M. I. T. Computation Center and Project MAC.

The majority of our programming has dealt with the solutions of transcendental and polynomial equations,[1, 2, 4, 5] least-squares fits[3] and Fourier analysis. Undoubtedly, this work will continue.

An increasing amount of our programming, however, has not had direct scientific application. One of our programs has helped to index a book.[6] Another is helping to prepare budgets for the Business Office of our Laboratory.

In addition to actual programming, we also provide other computer-oriented services. We consult with Laboratory members who wish to do their own programming.[7] We have given several demonstrations of the time-sharing system. At present, we are working in conjunction with Professor Sanborn C. Brown of the Plasma Physics Group and Mr. John H. Hewitt of our Document Room to set up our own files for use with the M.I.T. Library's information retrieval program, TIP. Sixty Laboratory members have signed up for our "Fortran IV" course.

Our work in both old and new areas will reflect the increasing needs of Laboratory members for computers as research aids.

Martha M. Pennell

References

1. Martha M. Pennell, "Newton's Method for Finding Complex Roots of a Transcendental Equation," Quarterly Progress Report No. 80, Research Laboratory of Electronics, M.I.T., January 15, 1966, pp. 263-266.

2. Martha M. Pennell, "Numerical Example to Illustrate Kizner's Method for Solving Nonlinear Equations," Quarterly Progress Report No. 80, Research Laboratory of Electronics, M.I.T., January 15, 1966, pp. 266-267.

3. Martha M. Pennell and Heather Davis, "Linearizing the Roots of a Polynomial," Quarterly Progress Report No. 83, Research Laboratory of Electronics, M.I.T., October 15, 1966, pp. 179-181.

4. Martha M. Pennell, "Further Computations Using Newton's Method for Finding Complex Roots of a Transcendental Equation," Quarterly Progress Report No. 81, Research Laboratory of Electronics, M.I.T., April 15, 1966, pp. 253-254.

5. Veronica E. McLoud and Martha M. Pennell, "Example of Symbolic Manipulation of Polynomials in MAD," Quarterly Progress Report No. 82, Research Laboratory of Electronics, M.I.T., April 15, 1966, pp. 294-295.

*This work was supported in part by the Joint Services Electronics Programs (U.S. Army, U.S. Navy, and U.S. Air Force) under Contract DA 36-039-AMC-03200(E).

6. Martha M. Pennell and R. M. Nacamuli, "A Computer Indexing Program," Quarterly Progress Report No. 82, Research Laboratory of Electronics, M.I.T., April 15, 1966, pp. 293-294.

7. M. G. Smith, "Computer Location of Plasma Dispersion Function Multiple Roots," S.M. Thesis, Department of Electrical Engineering, M.I.T., September 1966.

A. UPDATING OF THE COMPUTER INDEXING PROGRAM

In recent months, our basic computer indexing program has undergone marked revision. We feel that the program is now more versatile and hence more readily adaptable to the needs of a small research group.

The basic program permutes and alphabetizes the key words of up to 200 titles for a maximum of 78 characters per title. A second version of the program will do the same for a maximum of 70 titles, each consisting of no more than 132 characters.

In our basic version of the indexing program, the first 6 input characters may contain some sort of identification code, while characters 7-78 contain the title to be permuted. The output consists of the identification code in the first 6 columns, with the output title beginning in column 8. At present, the key word begins in the first permuted word of the output.

The program was originally designed to index upon key words of at least four letters. The disadvantages[1] of this scheme are obvious. Hence, a change was made in the program so that by use of the special character "$", key words of fewer than 4 letters will be indexed upon, and non-key words of more than 4 letters will be ignored. For example, suppose we are given the following input title:

23 ELASTIC COLLISION PROBABILITY FOR ELECTRONS IN ARGON (A)

Under normal circumstances, the key words would be: Elastic, Collision, Probability, Electrons, and Argon. The word "(A)" would be considered as a non-key word. By inserting the special character "$", however, we can generate the minimum desired number of permuted output titles. In our example, by judicious use of the "$", we can cause the phrase "Elastic Collision Probability" to be considered as a key word and also have "(A)" indexed as a key word:

23 ELASTIC$COLLISION$PROBABILITY FOR ELECTRONS IN ARGON (A)$

The "$" is blanked after the permutation for key words has been performed and before the output titles are alphabetized.

The program described here has been used to create an author and subject index in the updating of <u>Basic Data of Plasma Physics</u>[2] by Sanborn C. Brown. The punched output will be used to create listings suitable for photo-offset reproduction.

This work was done in part at the M.I.T. Computation Center.

Elaine C. Isaacs

References

1. Martha M. Pennell and R. M. Nacamuli, "A Computer Indexing Program," Quarterly Progress Report No. 82, Research Laboratory of Electronics, M.I.T., July 15, 1966, p. 23.

2. S. C. Brown, "Computer Programmed 'Basic Data of Plasma Physics'," Quarterly Progress Report No. 80, Research Laboratory of Electronics, M.I.T., January 15, 1966, pp. 83-85.

B. A SIMPLE METHOD FOR FINDING THE ROOTS OF AN ANALYTIC FUNCTION

Preliminary efforts have been made toward implementation of a simple procedure for finding the roots of an arbitrary complex function. The completed program will be especially useful in plasma physics research, in which exact solutions to dispersion equations are usually impossible, and numerical approximations are often valid only for a special type of equation. The method described in this report permits the user to input any analytic function, regardless of whether it happens to be algebraic.

Because the procedure relies on proper selection of curves for evaluation of a line integral, the facilities provided by the ESL Display Console at Project MAC can be profitably used. The techniques of man-machine interaction already developed there give the user a quick and intuitive way to deal with a complicated and formidable dispersion relation. The techniques that we plan to utilize are sketched in the program description below.

We want to find the roots of $f(z) = 0$, where $f(z)$ is given complex function. A simple consequence of Cauchy's residue theorem permits us to state that a root, Z_o, of $f(z)$ can be expressed[1] as

$$Z_o = \frac{1}{2\pi i} \oint_C \frac{zf'(z)}{f(z)} dz = \frac{1}{2\pi i} \oint_{f(C)} \frac{z}{f} df. \tag{1}$$

We are interested in the second form of this equation.

Determination of Z_o, using (1), requires the following computational procedures:

1. Display $f(z)$ in the z-plane and determine the existence of a root by inspection.
2. Enclose the root by a curve, C, using the typewriter or the light pen.
3. Compute and display $f(C)$ in the $w = f(z)$ plane. If $f(C)$ does not encircle the origin or encircles it more than once (indicating the presence of additional roots), return to (1) and enter C again.

4. Using Simpson's rule, evaluate the line integral

$$Z_o = \frac{1}{2\pi i} \oint_{f(C)} \frac{z}{f}\, df.$$

This approximation to Z_o is used to obtain a refined value of Z_o, using for C a circle centered at Z_o, and repeating 2) to 4) until a sufficiently precise value is determined. Previous work indicates that convergence is typically quite rapid.[1]

Eleanor C. River

References

1. B. D. Fried, "On Line Root Finding in the Complex Plane," Draft, 21 May 1966.

C. ROOTS, A ROOT-FINDING SUBROUTINE USING MULLER'S METHOD

A reliable polynomial-with-complex-coefficients-root-finding subroutine is the basis for such work as Quixot Dulcin[1] and Mills' display program.[2] In each case the same subroutine, called Roots, was used to perform this vital function. Roots is based on Share Distribution #692 which we modified to handle either a MAD or a Fortran calling program, using Muller's Method[3] to calculate the roots. Because Roots is extremely easy to use (the user need only provide the degree of the polynomial and its coefficients) and seldom fails to converge, it has been extremely useful.

Unfortunately, the program is coded in FAP (the assembly language for the IBM 7090-7094 computers), and consequently incompatible with the IBM 360 series to which the M.I.T. Computation Center will eventually be converting. Moreover, we have been plagued with numerous requests for a compiler language version of Roots. Consequently, we are rewriting Roots in both MAD and Fortran IV languages.

Because of its reliability, we have tried to follow the logic of the original Share Distribution, making modifications only when necessary to prevent cases in which Roots has been known to fail. This usually occurs when the constant term is eight or more orders of magnitude smaller than the other coefficients, i.e., there is a root extremely close to zero.

For example, Roots failed on the following polynomial:

$$x^4 + .076977968x^3 + .0014028739x^2 - .30980169 \times 10^{-5} x - .61770526 \times 10^{-19} = 0,$$

but succeeded on

$$x^4 + .076977968x^3 + .0014258634x^2 - .22131765 \times 10^{-5} x - .77008791 \times 10^{-9} = 0,$$

to give

$-.29337885 \times 10^{-3}$, $.0017075531$, $-.039196071 \pm .00094472884i$.

The method depends on three initial guesses which the program sets internally. These guesses, which remain the same for each polynomial, were not those originally suggested by Muller.

An important problem that we have encountered is round-off. The computer sometimes introduces a seemingly small imaginary or real part. The user is then left to ascertain whether this is a true contribution or whether the root is really pure real or imaginary. For example, on the polynomial whose roots are $\frac{1}{3}$, $\pm$ i (double root):

$$3x^5 - x^4 + 6x^3 - 2x^2 + 3x - 1 = 0 \tag{1}$$

our MAD program gave the following as roots:

$$.33333334$$

$$-.33913844 \times 10^{-13} - .99995337i$$

$$.80916850 \times 10^{-4} + .9999464i$$

$$-.43413214 \times 10^{-9} + 1.0000466i$$

$$-.80916414 \times 10^{-4} - 1.0000053i$$

Muller's method is a very easy technique both to understand and program. If we have three points, x_i, x_{i-1}, x_{i-2}, we can approximate the general polynomial:

$$f(x) = a_o x^n + a_1 x^{n-1} + \ldots + a_n = 0 \tag{2}$$

by the quadratic

$$b_o x^2 + b_1 x + b_2 = 0 \tag{3}$$

whose curve passes through the points $(x_i, f(x_i))$, $(x_{i-1}, f(x_{i-1}))$, $(x_{i-2}, f(x_{i-2}))$, and b_1, b_2, b_o satisfy the following system of equations:

$$b_o x_i^2 + b_1 x_i + b_2 = f(x_i)$$

$$b_o x_{i-1}^2 + b_1 x_{i-1} + b_2 = f(x_{i-1})$$

$$b_o x_{i-2}^2 + b_1 x_{i-2} + b_2 = f(x_{i-2}).$$

The quadratic formula easily gives us the roots of Eq. 3. Choosing as x_{i+1} that root of (3) closest in magnitude to x_i, we can again approximate Eq. 2 by Eq. 3, this time using the three points x_{i+1}, x_i, x_{i-1}. This process is repeated until

$$|x_{i+1}-x_i|/|x_{i+1}|$$

has become less than some preassigned number ($3.\times 10^{-8}$). The current value of x_i is then taken to be the root and divided into f(x) to obtain a new polynomial of degree $n-1$. This entire process is then repeated until a quadratic is formed, at which point the standard formula is used. The question naturally arises, What values does one use for the three initial points x_o, x_1, x_2?

In his article, Muller uses the starting values $x_o = -1$, $x_1 = 1$, $x_2 = 0$ and

$$a_n - a_{n-1} + a_{n+1} \qquad \text{for } f(x_o)$$

$$a_n + a_{n+1} + a_{n-2} \qquad \text{for } f(x_1)$$

$$a_n \qquad \text{for } f(x_2).$$

The rationale behind this choice is that Eq. 3 then becomes equal to the last three terms of the original equation

$$a_n + a_{n-1}x + a_{n-2}x^2$$

which is a good approximation if a root exists near zero. A second advantage is that this choice saves two evaluations of the original polynomial.

We found, however, that in the Share Distribution the true values for $f(x_o)$, $f(x_1)$, $f(x_2)$ were used. In fact, our first MAD program used Muller's starting guesses and failed on the following polynomial:

$$x^5 - 2x^4 - 6x^3 + 4x^2 + 13x + 6 = 0$$

whose roots are 2, 3, −1 (triple). The Share Distribution did not fail, however, nor did our MAD program when the true values for $f(x_o)$, $f(x_1)$, $f(x_2)$ were used. The same thing occurred on Eq. 1.

An investigation is now under way to determine the reason. Work also continues on debugging and testing our routines.

Martha M. Pennell, Joan Harwitt

References

1. Martha M. Pennell, "Generalized Polynomial Root-Finding Program for a Time-Shared Computer," Quarterly Progress Report No. 79, Research Laboratory of Electronics, M.I.T., October 15, 1965, pp. 263-266.

2. J. M. Mills, "A Computer Display System for Analyzing Polynomial-Type Dispersion Relations," S.M. Thesis, Department of Electrical Engineering, M.I.T., January 1965.

3. D. E. Muller, "A Method for Solving Algebraic Equations Using an Automatic Computer," Mathematical Tables and Other Aids to Computation (National Research Council, Washington, D.C., October 1956), pp. 208-215.

D. LINEARIZING THE ROOTS OF A POLYNOMIAL. II

Using the least-squares method as previously described,[1] we approximated the following sixth-degree polynomial in λ:

$$\begin{vmatrix} \Gamma_{11}-\lambda^2 & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12} & \Gamma_{22}-\lambda^2 & \Gamma_{23} \\ \Gamma_{13} & \Gamma_{23} & \Gamma_{33}-\lambda^2 \end{vmatrix} = 0$$

where

$$\Gamma_{11} = k_x^2 C_{11} + k_y^2 \frac{1}{2}(C_{11}-C_{12}) + k_z^2 C_{44} + 2k_y k_z C_{14}$$

$$\Gamma_{22} = k_x^2 \frac{1}{2}(C_{11}-C_{12}) + k_y^2 C_{11} + k_z^2 C_{44} - 2k_y k_z C_{14}$$

$$\Gamma_{33} = \left(k_x^2+k_y^2\right) C_{44} + k_z^2 C_{33}$$

$$\Gamma_{12} = 2k_x k_z C_{14} + \frac{1}{2} k_x k_y (C_{11}+C_{12})$$

$$\Gamma_{13} = k_x k_z (C_{44}+C_{13}) + 2k_x k_y C_{14}$$

$$\Gamma_{23} = \left(k_x^2-k_y^2\right) C_{14} + k_y k_z (C_{13}+C_{44})$$

$$k_x = \sin\theta \cos\gamma$$

$$k_y = \sin\theta \sin\gamma$$

$$k_z = \cos\theta$$

$$0 \leq \theta \leq 180^\circ \quad \text{and} \quad 0 \leq \gamma \leq 30^\circ$$

$C_{11} = .8694$	$C_{33} = 1.0680$	$C_{44} = .5762$
$C_{12} = .0696$	$C_{13} = .1560$	$C_{14} = .1743$

The problem is to find the set of constants $\phi_1, \ldots, \phi_{14}$ such that one of the following expressions best approximates the three positive real roots of the polynomial:

1) $\phi_1\omega_o + \phi_2\omega_2^{1}$

2) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4$

3) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3$

4) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6$

5) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3$

6) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6$

7) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8$

8) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8 + \phi_9\omega_8^3$

9) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8 + \phi_9\omega_8^3 + \phi_{10}\omega_8^6$

10) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8 + \phi_9\omega_8^3 + \phi_{10}\omega_8^6 + \phi_{11}\omega_{10}$

11) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8 + \phi_9\omega_8^3 + \phi_{10}\omega_8^6 + \phi_{11}\omega_{10}$
$+ \phi_{12}\omega_{10}^3$

12) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8 + \phi_9\omega_8^3 + \phi_{10}\omega_8^6 + \phi_{11}\omega_{10}$
$+ \phi_{12}\omega_{10}^3 + \phi_{13}\omega_{10}^6$

13) $\phi_1\omega_o + \phi_2\omega_2 + \phi_3\omega_4 + \phi_4\omega_4^3 + \phi_5\omega_6 + \phi_6\omega_6^3 + \phi_7\omega_6^6 + \phi_8\omega_8 + \phi_9\omega_8^3 + \phi_{10}\omega_8^6 + \phi_{11}\omega_{10}$
$+ \phi_{12}\omega_{10}^3 + \phi_{13}\omega_{10}^6 + \phi_{14}\omega_{10}^9$

$$\omega_o = 1$$

$$\omega_2 = \frac{1}{2}(3\cos^2\theta - 1)$$

$$\omega_4 = \frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$$

$$\omega_4^3 = (-1)\ 105 \sin^3\theta\cos\theta\sin 3\gamma$$

$$\omega_6 = (-1)\frac{1}{16}(5 - 105\cos^2\theta + 315\cos^4\theta - 231\cos^6\theta)$$

$$\omega_6^3 = \left(\frac{945}{2}\cos\theta - \frac{3465}{2}\cos^3\theta\right)\sin^3\theta\sin 3\gamma$$

$$\omega_6^6 = \frac{1}{16}(231\sin^6\theta\sin 6\gamma)$$

$$\omega_8 = \frac{1}{128}(35 - 1260\cos^2\theta + 6930\cos^4\theta - 12012\cos^6\theta + 6435\cos^8\theta)$$

$$\omega_8^3 = (-1)\frac{1}{128}\left(6930\cdot 4!\cos\theta - 2002\cdot 6!\cos^3\theta - \frac{429}{8}\cdot 8!\cos^5\theta\right)\sin^3\theta\sin 3\gamma$$

$$\omega_8^6 = (-1)\frac{1}{128}\left(12012\cdot 6! - \frac{6435}{2}\cdot 8!\cos^2\theta\right)\sin^6\theta\sin 6\gamma$$

$$\omega_{10} = (-1)\frac{63}{256}\left(1 - 55\cos^2\theta + \frac{1430}{3}\cos^4\theta - 1430\cos^6\theta + \frac{12155}{7}\cos^8\theta - \frac{46189}{63}\cos^{10}\theta\right)$$

Table XXXIV-1. Results.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	ϕ_{12}	ϕ_{13}	ϕ_{14}	Standard Deviation	R^2	F-Ratio	De F
Middle Curve																		
Coefficient	.72523	.03456	.07714	.00267	-.07197										.02030	.9120	165.86	
Standard Error	(.00285)	(.00672)	(.00822)	(.00012)	(.00982)													
-value	254.37	5.14	9.38	2.27	-7.33													
Coefficient	.72643	.03725	.06914	-1.07592	-.06686	-.54987	-.879E-6	.00649	-.15134	-.195E-6	-.00932	-.01784	-.165E-7	-.588E-10	.01744	.9410	67.52	
Standard Error	(.00388)	(.00838)	(.01082)	(.56803)	(.01142)	(.28961)	(.102E-5)	(.01360)	(.07973)	(.105E-7)	(.00959)	(.00939)	(.304E-7)	(.315E-10)				
-value	187.37	4.44	6.39	-1.89	-5.85	-1.90	.86	.48	-1.90	-1.85	-.97	-1.90	-.54	-1.87				
Lower Curve																		
Coefficient	.60904	.03261	.03980	.00208	.06688	-.00027									.01824	.9067	122.52	
Standard Error	(.00261)	(.00604)	(.00739)	(.00011)	(.00888)	(.00005)												
-value	233.40	5.40	5.38	19.20	7.53	-5.18												
Coefficient	.61241	.02393	.04509	.92991	.06949	.47278	-.196E-5	-.00456	.13021	.694E-7	.00825	.01534	.185E-7	.305E-10	.01756	.9214	49.60	
Standard Error	(.00390)	(.00844)	(.01089)	(.57196)	(.01150)	(.29161)	(.103E-5)	(.01369)	(.08028)	(.106E-6)	(.00966)	(.00945)	(.306E-7)	(.317E-10)				
-value	156.88	2.84	4.14	1.62	6.04	1.62	1.90	-.33	1.62	.65	.85	1.62	.60	.96				
Outer Curve																		
Coefficient	1.0338	.07636	-.07818	-.00310											.01827	.9375	325.23	
Standard Error	(.00228)	(.00605)	(.00739)	(.00010)														
-value	452.79	12.63	-10.58	-29.40														
Coefficient	1.02612	.08434	-.07491	.29992	.00280	.15462	.105E-5	-.00650	.04248	.123E-6	.000430	.00501	-.257E-8	.588E-10	.01714	.9532	86.15	
Standard Error	(.00381)	(.00824)	(.01063)	(.55837)	(.01123)	(.28468)	(.100E-5)	(.01337)	(.07838)	(.104E-6)	(.00943)	(.00923)	(.297E-7)	(.310E-10)				
-value	269.25	10.24	-7.04	.54	.25	.54	1.05	-.49	.54	1.18	.04	.54	-.09	1.89				

$$\omega_{10}^{3} = \frac{63}{256}\left(\frac{1430}{3}\cdot 4!\cos\theta - \frac{715}{3}\cdot 6!\cos^{3}\theta + 12155\cdot 42\cos^{5}\theta - \frac{46189}{63}\cos^{7}\theta\right)\sin^{3}\theta\sin 3\gamma$$

$$\omega_{10}^{6} = \frac{63}{256}\left(1430\cdot 6! - \frac{12155}{14}\cdot 8!\cos^{2}\theta + \frac{46189}{63}\cdot\frac{10!}{4!}\cos^{4}\theta\right)\sin^{6}\theta\sin 6\gamma$$

$$\omega_{10}^{9} = (-1)\frac{1}{256}\cdot 46186\cdot 10!\cos\theta\sin^{9}\theta\sin 9\gamma$$

n arbitrary roots on the same roots loci were used as data for the least-squares analysis. Because of the nature of the curves and the number of fitting functions, we repeated our calculations with 30, 52, 70, and 90 data points. We found that 30 points were too few and gave spurious results for fitting functions 10 through 13.

We used the following criteria to measure the "goodness" of fit:

R^2 (also called the coefficient of correlation): May be looked upon as the proportion of total "squared error" that is explained by fitting the line as "due to" the linear relationship between the dependent variable and independent variables.[3] A perfect relationship is 1 and no relationship is 0.

Standard Deviation: The measure of dispersion around the fitted line. Two-thirds of the observed points are within one standard deviation of each side of the line.[4]

F-ratio: The explained variation over the unexplained variation of the dependent variables about the least-squares line.[5] For a sample with 37 degrees of freedom and 14 coefficients, 1.89 indicates a good fit; for 55 degrees of freedom and 14 coefficients, 1.88; and for 77 degrees of freedom and 14 coefficients, 1.82. For all equations in our samples, the F-ratio was significant except for the first fitting function.[6]

Standard Error of the coefficient: Similar to the standard deviation of the equation.

t-value: Similar to the F-ratio but is used to test the level of significance of the coefficient rather than the whole equation. At a confidence level of .90, a t-value of 1.645 was significant; at the .95 level the t-value must be 1.960; at the .98 level, 2.326; and at the .99 level, 2.576.[3]

The formulas for calculating these statistical measures are taken from Econometric Models.[7]

The results are summarized in Table XXXIV-1 for the three curves with 70 points used. The fitting functions given here have the best fit as indicated either by the largest F-ratio or the largest R^2 and the smallest standard deviation. The results of the other samples are comparable.

Heather S. Davis

References

1. Martha M. Pennell and Heather Davis, "Linearizing the Roots of a Polynomial," Quarterly Progress Report No. 83, Research Laboratory of Electronics, M.I.T., October 15, 1966, pp. 179-181.

2. G. W. Farnell, "Elastic Waves in Trigonal Chrystals," Can. J. Phys. 39, 65-80 (January 1961).

3. B. L. Nelson, Elements of Modern Statistics (Appleton-Century-Crofts, Inc., New York, 1958), pp. 242-243; see t-value table, p. 341.

4. John F. Kenney, Mathematics of Statistics, Part I (D. Van Nostrand Company, New York, 2d edition, 1961), pp. 68-69.

5. J. Johnston, Econometric Methods (McGraw-Hill Book Company, New York, 1963), pp. 32-33.

6. C. R. C. Standard Mathematical Tables (Student Edition, 1964), pp. 261-264.

7. J. Johnston, op. cit., pp. 134-135.

E. DEVELOPMENT OF PRIVATE LIBRARY TO BE SEARCHED BY TIP

On the initiative of Professor Sanborn C. Brown, a project is under way to create a series of private user files[2] on Project MAC to be searched by the M.I.T. Library TIP Program. The four libraries now being catalogued are those of Professors Allis, Brown, Bers, and Bekefi – all of whom work in the field of Plasma Physics. This project draws extensively upon the facility to create users' files as programmed in the M.I.T. Technical Information Project[1] and upon the user interaction and time-sharing capabilities of Project MAC.

The files are created and maintained by each user or his secretary, except for Professor Allis' whose file is maintained by the Document Room staff of the Research Laboratory of Electronics. Each file contains one entry corresponding to each report, book, etc. in the individual's library. In turn each entry contains the information needed to locate the article, such as the title (unabbreviated), author(s) and source, as well as a code incorporating the Professor's initials to indicate where the publication can be found. Following are examples of entries from each of these four files.

20A 125 - **BROWN**

THE PLASMA PHYSICS OF THERMIONIC CONVERTERS
R. H. BULLIS, L. K. HANSEN, J. M. HOUSTON,
M. F. KOSKINEN, N. S. RASOR, C. WARNER
THERM CONV SPEC CONF SAN DIEGO 1965 SCE 50

ANALYSIS OF BIBLIOGRAPHIC SOURCES IN A GROUP OF PHYSICS-RELATED JOURNALS
M. M. KESSLER
M.I.T. SCE 83

PHYSIC SURVEY AND OUTLOOK
NAT. ACAD. OF SCIENCES
WASHINGTON SCE 89

TRANSACTIONS OF THE ALL-UNION CONF. ON SPACE PHYSICS
G. A. SKURIDIN
NASA TT F-389 SCE 102

PLASMA PHYSICS BULLETIN
DEPT. OF PHYSICS
U. OF MIAMI V3 1963 SCE 287

PROCEEDINGS FAR INFRARED PHYSICS SYMPOSIUM
NAVAL ORDANCE LAB. CORONA
CORONA, CAL. SCE 316

PHYSICS OF FLUIDS
A.I.P.
V9 P1437 1966 SCE 359

20E 120 - **BERS**

PLASMA PHYSICS LABORATORY REVIEW
J. E. DRUMMOND ET AL
BOEING JAN-JUN 1965 AE 14

MECHANICAL DESIGN FOR PLASMA PHYSICS RESEARCH
W. S. NEEF, JR.
UCRL 12325 JAN 1965 AE 79

AN ANNOTATED BIBLIOGRAPHY OF ARTICLES ON PLASMA PHYSICS AND CONTROLLED THERMONUCLEAR RESEARCH BY U.K.A.E.A. STAFF, 1958 TO 1962
L. J. ANTHONY
CLM R 24 JAN 1963 AE 118

20A 112 - **BEKEFI**

REPORT OF 1965 NAGOYA MEETING ON SPACE PLASMA PHYSICS
NAGOYA UNIVERSITY IPPJ-44(J) DEC. 1965 GE 5
H. IKEZI AND K. TAKAYAMA
NAGOYA UNIVERSITY IPPJ-48 MARCH 1966 GE 6

PLASMA PHYSICS LABORATORY REVIEW, JAN.-JUNE 1966
SCIENTIFIC RESEARCH LABORATORIES
BOEING, GE 22

26 237 - **ALLIS**

PLASMA PHYSICS LABORATORY REVIEW
J. E. DRUMMOND, ET AL
BOEING SCIENTIFIC RES LAB JULY-DEC 1965 WPA 10

PHYSIQUE DES FAISCEAUX DE PLASMA DE SYNTHESE
J. F. BONNAL, J. GIACOMINI, G. MAINFRAY, C. MANUS,
J. MORELLEC, G. SPIESS
SACLAY SEPT 6-10, 1965 WPA 19

CUMULATIVE LISTING OF PLASMA PHYSICS LABORATORY REPORTS AND PUBLICATIONS (5TH EDITION)
PRINCETON UNIV
PRINCETON UNIV MATT-400 JULY 1966 WPA 48

As we show in the examples above, the file code name indicates the room where the publication can be found.

By using the information retrieval services of the Library TIP Program[2] and Project MAC, these four files may be searched individually or all at once; the latter procedure is facilitated by linking to PRIVAT REPORT in Professor S. C. Brown's files.

We hope that this project will cut down the time and effort spent in organizing and using these private libraries and reduce the number of duplicates. It should make it easier to share information. At the present time, Professor Brown has 466 articles listed; Professor Allis, 52; Professor Bers, 124; and Professor Bekefi, 18.

Heather S. Davis

References

1. M. M. Kessler, Physics Today, Vol. 18, No. 3, pp. 28-36, March 1965.
2. M. M. Kessler, TIP User's Manual, December 1, 1965.